

Data Collection Activity - TICTOC

Objective: In this activity we are going to represent the motion of a pendulum with a sinusoidal function. We will then apply some calculus concepts to this function and its graph to further understand topics such as velocity, acceleration, etc.

Building the Function

The data we have collected and graphed is a distance (in L2) vs. time (in L1) scatterplot of the racquetball as it swings back and forth. It appears to produce a sinusoidal pattern, so we will attempt to fit our data with the sinusoidal function $d(t)$ in the form:

$$d(t) = A \cos(B(t - C)) + D$$

We need to find values for A, B, C, and D. (If necessary, round all values to 2 decimal places.)

1. Find the value of C first. Since we are using a cosine function, trace to the point on the scatterplot where the value of C can be found and record it.

$$C = \underline{\hspace{2cm}}$$

2. Now find the value of D. This should be the **average** of the "highest value" on the scatterplot and the "lowest value". By tracing, find these values, and show the arithmetic used to find D.

$$D = \underline{\hspace{2cm}}$$

3. Now find the value of A. You already have all of the information to find A.

$$A = \underline{\hspace{2cm}}$$

4. Finally, find the value of B. How many periods of the cosine function are on your scatterplot?
To find an accurate value for B, trace to the initial point of one period of the cosine function (which, actually, is the value of C) and then to the final point of the last period of the function. You should be able to find one period of the function if you subtract these two values and divide by the number of periods.

$$\text{Length of one period: } \underline{\hspace{2cm}}$$

Now you can find B. (Don't forget you need to use the number 2π to find B.) Record the value of B.

$$B = \underline{\hspace{2cm}}$$

5. So, record the final cosine function $d(t)$ that fits your data.

$$d(t) = \underline{\hspace{2cm}}$$

Enter this function into Y1 of your calculator and graph it. If it doesn't fit the data, find your mistake!

Velocity and Acceleration Functions

6. We also have the "velocity" data in L3 of our calculator. Set up and graph a scatterplot of the velocity vs. time data. This scatterplot shows the velocity of the racquetball at a specific time as it swings back and forth.

7. We could determine this function using the same procedure as we did for the distance vs. time data above, but we know calculus!

8. Determine the velocity function $v(t)$ with calculus.

$$v(t) =$$

9. We also have the “acceleration” data in L4 of our calculator, but, from experience, we know that this might not be very accurate. Set up and graph a scatterplot of the acceleration vs. time data.

10. Again, we can find a function that fits this data using calculus. Determine the acceleration function $a(t)$ with calculus.

$$a(t) = \frac{1}{\sqrt{1 - \frac{1}{4}t^2}}$$

Check your answer by graphing it with the scatterplot. If it doesn't fit the data, find your mistake!

Questions

11. Turn off your scatterplots. Set the window of your calculator so you are seeing **exactly two periods** of the velocity and acceleration functions. Sketch the two periods of $v(t)$ and $a(t)$ below. Label the graphs and label the important points.

Let's use the graphs to answer some questions about the pendulum.

12. Over what time intervals is the pendulum moving toward the CBR? _____

13. When is the pendulum moving the fastest? $t =$ _____

14. What is occurring on the acceleration function at these points where the pendulum is moving the fastest?

The Speed Function

15. The speed of the pendulum can be represented by the absolute value of the velocity function. Enter a function for $|v(t)|$ and graph it. (Note: You may want to temporarily deactivate $v(t)$ and $a(t)$).

16. What an interesting graph! The pendulum is speeding up when the “speed function” is increasing. Over what time intervals is the pendulum speeding up?

17. Now, let's see what the velocity function and the acceleration functions are doing on these time intervals. Deactivate the speed function and activate the velocity and acceleration functions. Rewrite the intervals above and tell whether the velocity and acceleration functions are positive (+) or negative (-) at these times.

Interval: _____

Velocity: _____

Acceleration: _____

Interval: _____

Velocity: _____

Acceleration: _____

Interval: _____

Velocity: _____

Acceleration: _____

18. The pendulum is slowing down when the "speed function" is decreasing. Activate the "speed function" again. Over what time intervals is the pendulum slowing down?

19. Deactivate the speed function and activate the velocity and acceleration functions one more time. Rewrite the intervals above and tell whether the velocity and acceleration functions are positive or negative at these times.

Interval: _____	Velocity: _____	Acceleration: _____
Interval: _____	Velocity: _____	Acceleration: _____
Interval: _____	Velocity: _____	Acceleration: _____
Interval: _____	Velocity: _____	Acceleration: _____

What are the patterns that you see?

A Bonus Problem!

20. Finally, let's look at one more relationship. Set up and graph a scatterplot between the "distance" data in L2, and the "velocity" data in L3. This result will be a surprise!

21. This type of relationship was discussed in class. By tracing (and using a little arithmetic), find the model that fits this data. To see if it is correct, solve for y in terms of x and graph it with the scatterplot. Show all of your work below. If it doesn't fit the data, find your mistake!